1.2 Imaginary & Complex Numbers

SWBAT simplify expression with imaginary numbers.

Imaginary Numbers

Until now you have been told you cannot take the square root of a negative number. Now, however, you can take the square root of a negative number, but it involves a new number called "i" which is called the imaginary number.

$$i = \sqrt{-1}$$

Simplify:

Ex. $\sqrt{-25}$

$i\sqrt{25}$

$\boxed{5i}$

Ex. $\sqrt{-9}$

$i\sqrt{9}$

$\boxed{3i}$

Ex. $\sqrt{-40}$

$i\sqrt{40}$

4 10

2 2 2 5

$\boxed{2i\sqrt{10}}$

Ex. $\sqrt{-125x^2}$

$i\sqrt{125 x x}$

25 5

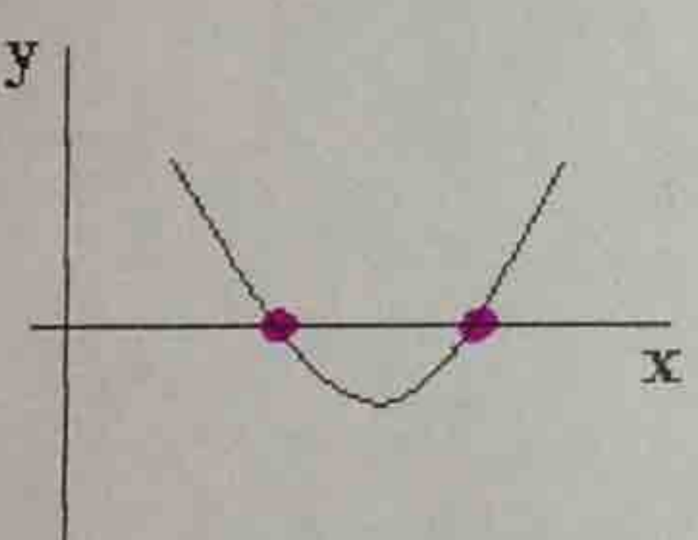
5 5

$\boxed{5xi\sqrt{5}}$

Where have we seen imaginary numbers before?

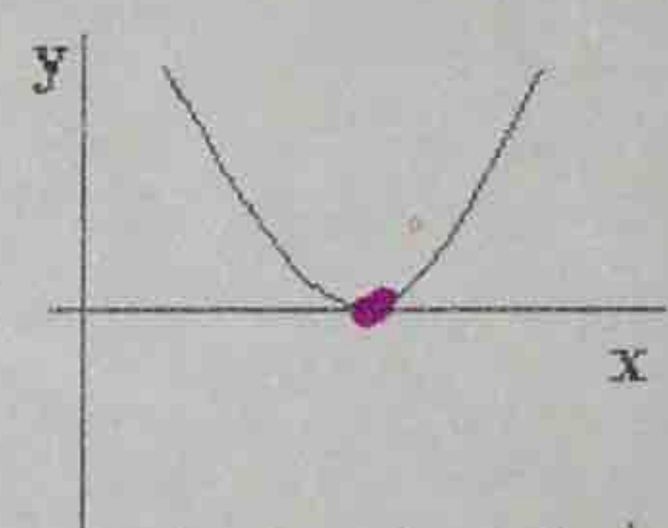
In Math 1, we learned the quadratic formula and the discriminant. The discriminant, $b^2 - 4ac$, determines the number of solutions and the type of solutions we will have with a quadratic equation.

Two Real
$b^2 - 4ac > 0$



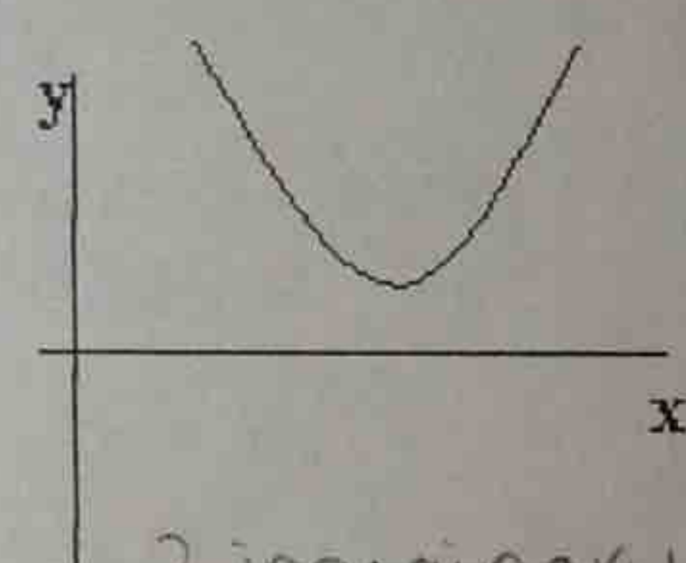
2 real solutions

One Real
$b^2 - 4ac = 0$



1 real solution
(double root)

No Real
$b^2 - 4ac < 0$



2 imaginary solutions

Simplifying Powers of i's

To simplify i to any power, try to get the exponent to an even power by removing an i if the exponent is odd, and then reverse the "power to a power" rule by dividing by two. Simplify using the properties of algebra.

Ex. $i^{17} =$

$= i \cdot i^{16}$

$= i \cdot (i^2)^8$

$= i(-1)^8$

$= i(1)$

$\boxed{= i}$

Ex. $i^{98} =$

$= (i^2)^{49}$

$= (-1)^{49}$

$\boxed{= -1}$

Ex. $i^{39} =$

$= i \cdot i^{38}$

$= i \cdot (i^2)^{19}$

$= i \cdot (-1)^{19}$

$= i(-1)$

$\boxed{= -i}$

Ex. $i^{65} =$

$= i \cdot i^{64}$

$= i \cdot (i^2)^{32}$

$= i \cdot (-1)^{32}$

$= i(1)$

$\boxed{= i}$

Imaginary Numbers: For any positive b, $\sqrt{-b^2} = \sqrt{b^2} \bullet \sqrt{-1}$

Example 1: Simplify $2\sqrt{-12} \cdot 3\sqrt{-3}$.

$2i\sqrt{12} \cdot 3i\sqrt{3}$

$6i^2\sqrt{36}$

$6i^2(6)$

$= 36i^2$

$\boxed{= -36}$

You Try! Simplify $\sqrt{-8} \cdot \sqrt{-32}$

$i\sqrt{8} \cdot i\sqrt{32}$

$i^2\sqrt{256}$

$16i^2$

$\boxed{= -16}$

Complex Numbers: What is a complex number?

$a + bi$

Real | imaginary

The real part is ALWAYS first!

Example 2: Name the real and imaginary part of

$7 + 4i$

real imaginary

Adding and Subtracting Complex Numbers: Only combine like terms. Double check with your calculator.

a) Simplify $(6-4i)+(1+3i)$

$6 - 4i + 1 + 3i$

$\boxed{7 - 1i}$

b) Simplify $(4-6i)-(3-7i)$

$4 - 6i - 3 + 7i$

$\boxed{1 - i}$

Let x and y be real numbers. What are the values of x and y?

a) $(x + yi) - (7 - 3i) = 12 + 9i$

$x + yi - 7 + 3i = 12 + 9i$

$+7 - 3i \qquad +7 - 3i$

$x + yi = 19 + 6i$

$\boxed{x = 19, \; y = 6}$

b) $(x + yi) + (9 - 4i) = -3 - 14i$

$x + yi + 9 - 4i = -3 - 14i$

$-9 + 4i \qquad -9 + 4i$

$x + yi = -12 - 10i$

$\boxed{x = -12, \; y = -10}$

Multiplying Complex Numbers: Make sure to FOIL. Double check with your calculator.

a) Simplify $(6-4i)(1+3i)$

$6 + 18i - 4i - 12i^2$

$6 + 14i + 12$

$\boxed{18 + 14i}$

b) Simplify $(4-6i)(3-7i)$

$12 - 28i - 18i + 42i^2$

$12 - 46i - 42$

$\boxed{-30 - 46i}$

Dividing Complex Numbers: Imaginary numbers may NEVER be in the denominator. To simplify, multiply the complex numbers by the conjugate (just like with radicals).

a) Simplify $\frac{3i}{(2+4i)} \cdot \frac{(2-4i)}{(2-4i)}$

$\frac{3i(2-4i)}{(2+4i)(2-4i)} = \frac{6i - 12i^2}{4 - 8i + 8i - 16i^2}$

$= \frac{12 + 6i}{4 + 16} = \frac{12 + 6i}{20}$

$= \boxed{\frac{6 + 3i}{10}}$

b) Simplify $\frac{3i}{(6-5i)} \cdot \frac{(6+5i)}{(6+5i)}$

$\frac{3i(6+5i)}{(6-5i)(6+5i)} = \frac{18i + 15i^2}{36 - 30i + 30i - 25i^2}$

$= \frac{-15 + 18i}{36 + 25} = \boxed{\frac{-15 + 18i}{61}}$